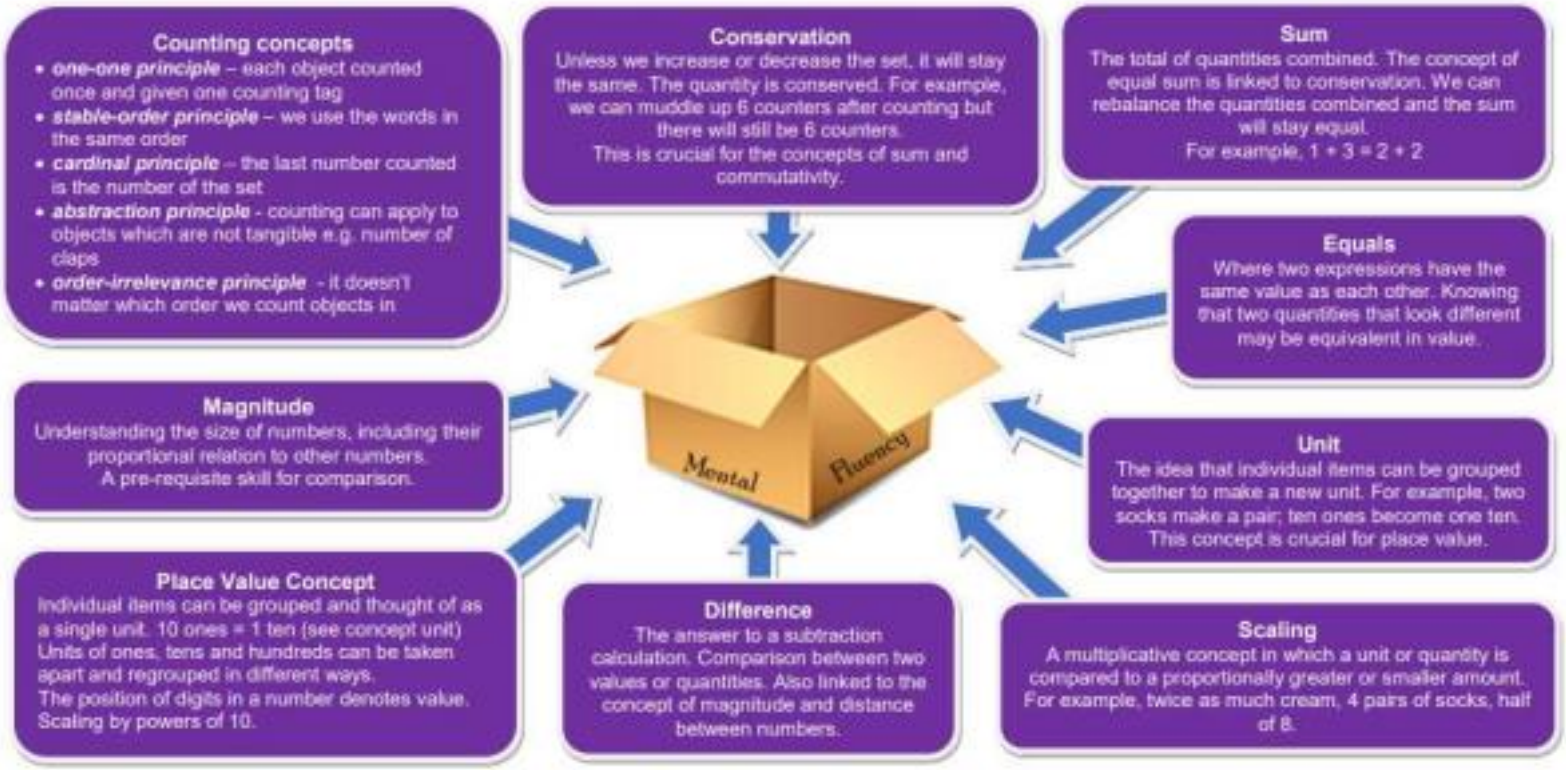
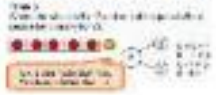
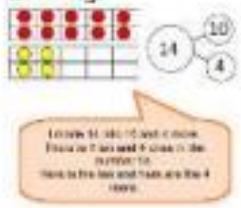
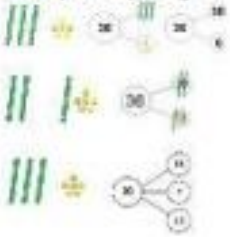
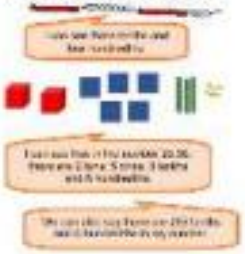
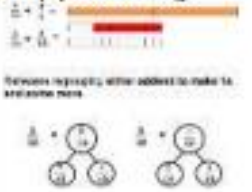
Fluency Skills Progression

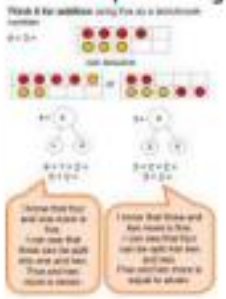
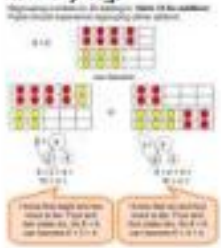
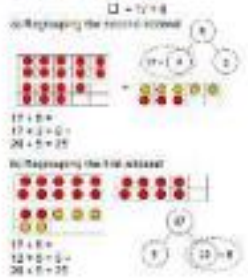
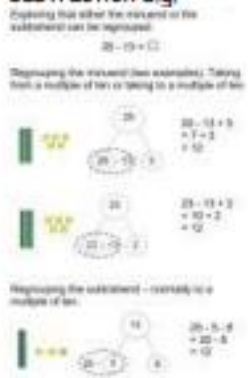
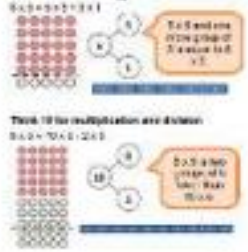
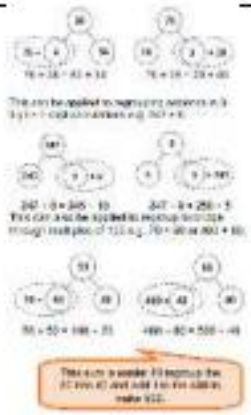
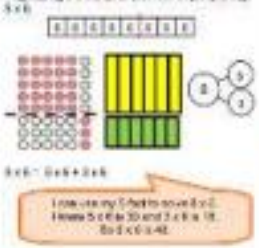
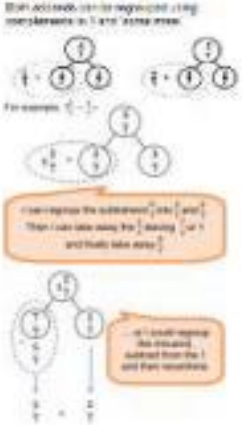
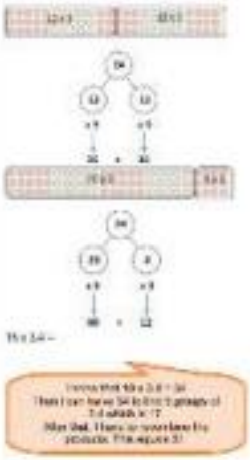
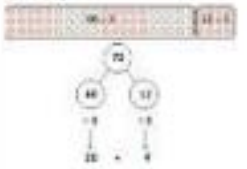
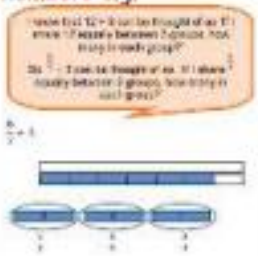
The NCETM set out 5 big ideas for mastery. These include coherence, representations, variation, mathematical thinking and fluency. They state that fluency demands more of students than memorisation of a single procedure or collection of facts. It encompasses a mixture of efficiency, accuracy and flexibility. Quick and efficient recall of facts and procedures is important in order for students to keep track of sub-problems, think strategically and solve problems. Fluency also demands the flexibility to move between different contexts and representations of mathematics, to recognise relationships and make connections, and to make appropriate choices from a whole toolkit of methods, strategies and approaches. This document aims to provide a structure in which teachers will explicitly teach these in a coherent and well thought out manner. The core mathematical concepts children need to become fluent are:

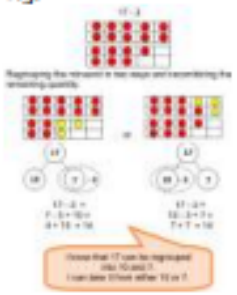
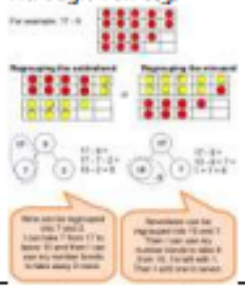
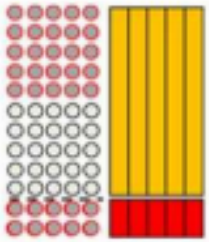
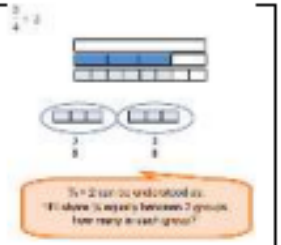
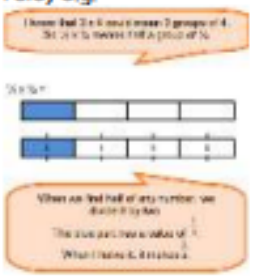
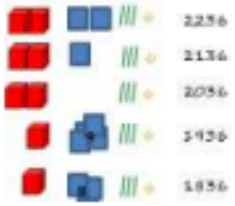


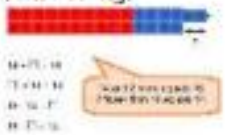
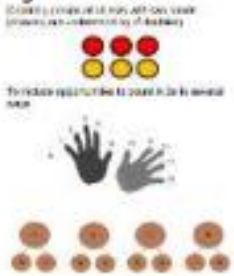
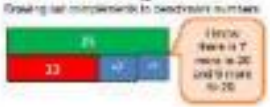
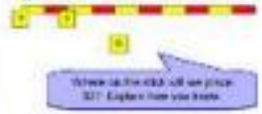
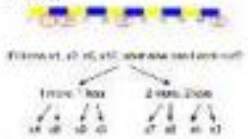
Subitising	the ability to see number as pattern, such as dice patterns. This supports pupils to see numbers within numbers and better regrouping (partitioning).
Regrouping (partitioning)	the ability to break numbers up and recombine them flexibly
Counting on and counting back	in a variety of interval steps
Reordering	knowing when and how to reorder to make calculations easier
Finding complements	links to reordering, identifying useful complements pairs or trios of 1, 10, 60 etc.
Applying the inverse	use of fact family knowledge to 'undo'
Rounding	to a range of benchmark numbers
Estimation	both linear estimation on number lines and scales, and of quantities and calculations to support an increasing sense of what is reasonable
Compensation	to use rounding to add or subtract too much or too little and adjust accordingly
Rebalancing	to adjust the parts of addition and subtraction facts to make a calculation easier
x ÷ by powers of 10	
Doubling and halving	
Rearranging	to adjust the groups in multiplication and division to make a calculation easier

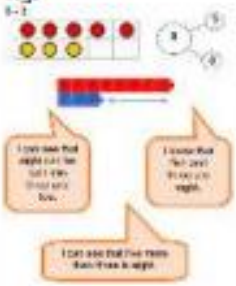
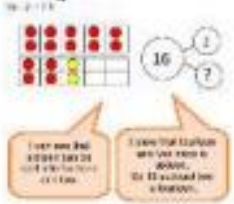
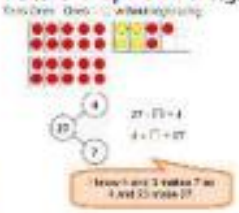
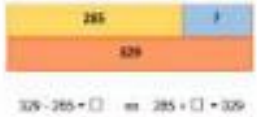
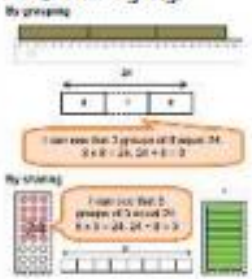
These are broken down below into skills which each year group should focus on.

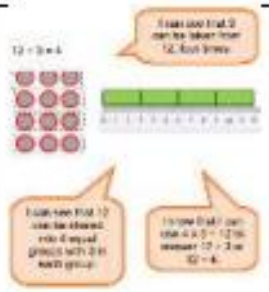
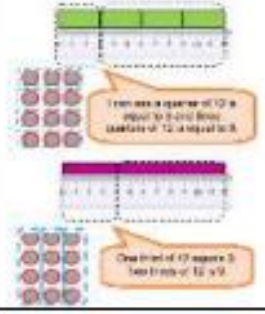
Fluency skill	F2	1	2	3	4	5 and 6
Subitising	Recognise groups up to 5 without the need to count Identify 5 and use as a benchmark number e.g.  Identify numbers within a whole set e.g.  I can see 3 and 3 and 1. I can see 4 and 1 and 1 and 1. I can see 4 and 3.	Recognise groups (up to 5) without the need to count and use this to identify numbers up to 10 				
Regrouping (partitioning)	Use 5 as a benchmark number  Understand a whole = part and part  Identify a group as a unit e.g. a unit is 2 teddies 	'Think 10' - able to partition a number into 10 and some more e.g.  Able to regroup a whole up to 10 into different parts and understand the commutativity of this	Regroup two digit numbers flexibly and in multiple ways e.g. 	Regroup three digit numbers flexibly and multiple ways e.g.  Regroup for addition to allow bridging through 10 and 100. Work flexibly and reason about the most efficient methods e.g.	Regroup four digit numbers flexibly and in multiple ways Regroup tenths and hundredths flexibly and in multiple ways e.g.  In addition, regroup to 'think 100' with more	Use regrouping as a valid method in a multi-step problem e.g.  Regroup for multiplication in a variety of ways e.g. 24

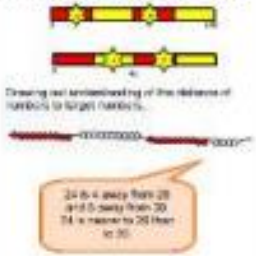
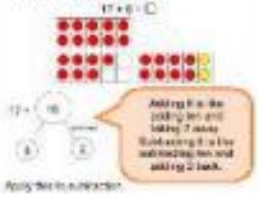
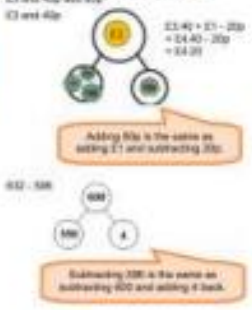
		Able to use 5 as a benchmark for regrouping for addition up to 10 e.g.  Able to use 10 as a benchmark for regrouping for addition up to 20 (think "10 and some more") e.g.  Able to use 10 as a benchmark for regrouping for subtraction up to 20, using wither the minuend or the subtrahend, without bridging through ten	Able to 'think 10' for addition e.g.  Able to 'think 10' for subtraction e.g.  'Think 5' for multiplication and division e.g. 	 Regroup for subtraction to allow bridging through 10 and 100. Work flexibly with both the minuend and the subtrahend. In multiplication, regroup to the '5' benchmark, working flexibly to regroup either the multiplier or the multiplicand e.g.  In multiplication, regroup to the 10 benchmark working flexibly to regroup	complex 3 digit addends e.g.  In subtraction, regroup to 'think 100' with more complex 3 digit numbers Use regrouping strategies to fractional part wholes, as well as other measures such as time and money e.g.  Use regrouping methods to find unknown products in multiplication e.g. 	x 3  Regroup for division in a variety of ways e.g. 72 ÷ 3  Regroup for division of fractions by whole numbers e.g. 

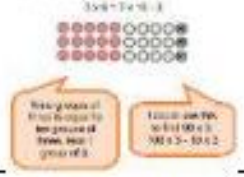
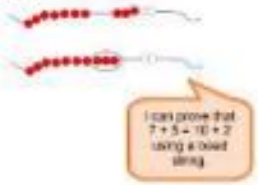
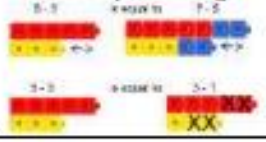
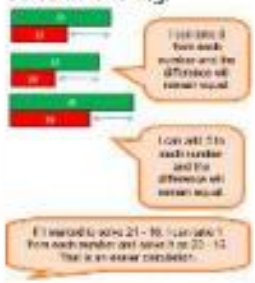
		e.g.  Able to use 10 as a benchmark for regrouping for subtraction up to 20, using either the minuend or the subtrahend, bridging through ten e.g. 		either the multiplier or the multiplicand e.g. 12 x 5  12 x 5 = 10 x 5 + 2 x 5		 Regroup fractions for multiplication (focus on this understanding before applying the rule) e.g. 
Counting on and back	Be able to count fluently. To do this, children need to have mastered: The one-one principle – each object is counted once and given one counting tag The stable-order principle –we use the words in the same order	Count on to find the total and difference e.g.  Count on to find the total and difference and link this knowledge to fact	Count in units where units are different e.g. 36 = ten, twenty, thirty, one, two, three, four, five, six Count on and back from any two digit number and notice what changes	Count on and back from any three digit number and notice what changes and what doesn't e.g. 	Count on and back from any four digit number and notice what changes and what doesn't e.g.  Count on and back in multiples and make	

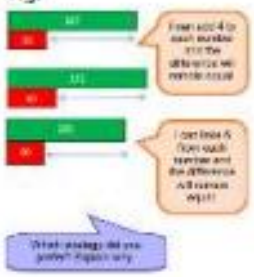
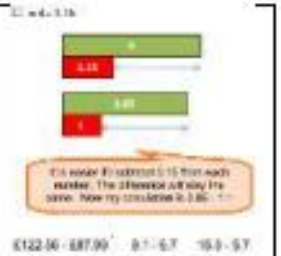
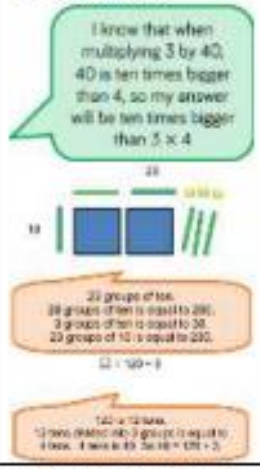
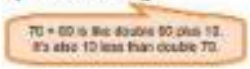
	The cardinal principle – the last number counted is the number of the set The abstraction principle – counting can apply to objects which are not tangible e.g. number of claps ▯ The order-irrelevance principle – it doesn't matter which order we count objects in Be able to count on when an addend is given, rather than count all Be able to identify the largest number and count on from it	families e.g.  Be able to skip count in a variety of ways to promote the 2x tables and doubles e.g.  Be able to skip count in 5s and 10s in a variety of ways, forwards, backwards and from different starting points.	and doesn't e.g.  Skip count in 3's in a variety of ways, forwards, backwards and from different starting points. Count on to find complements to benchmark numbers within 100 e.g. 	Count on to find complements to benchmark numbers within 1000 e.g.  Understand the connections between the 3, 4 and 8 times tables and know strategies to be able to work out unknowns e.g. 	counting connections e.g. counting in 6's, 60's, 600's, 0.6's Count in 25's, 50's, 0.1's and 0.001's Skip count all the times tables and understand the connections between them 	
Reordering and finding complements		Reorder to ensure efficient counting e.g. 	Reorder numbers to find complements when adding three one digit numbers e.g. 6+7+4 (add the 6 and 4 first to make 10)	Reorder three or more numbers up to 1000 to find complements e.g. 75+95+25 can be reordered into 75+25 to make the benchmark 100, then add 95 to total 195. Or 6+9+4+5+1 (reorder to make number bonds to 10)	Reorder three or more numbers up to 10,000 to find complements e.g. 800+240+360 310+700+30 = Reorder three or more numbers involving tenths and hundredths to find complements e.g. 1.5+3+0.5	Reorder three or more numbers to find complements where the arrangement is more complex e.g. £3.99+£7.80+£2.01

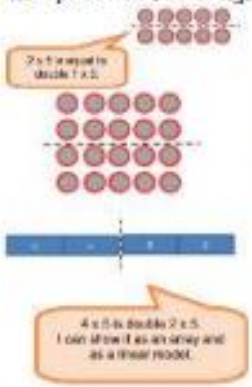
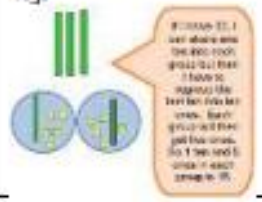
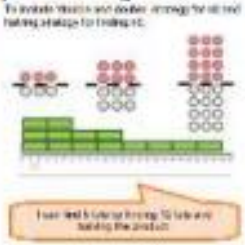
					2.5+25+5+2.5= (reorder to make wholes)	
Applying the inverse		Think addition to solve subtraction with numbers to 10 e.g.  Think addition to solve subtraction with numbers up to 20 e.g. 	Think addition to solve subtraction with numbers up to 100 e.g.  Understand the relationship between multiplication and division. In multiplication, understand the interrelationship between multiplier, multiplicand and product. In division, understand the relationship between the dividend, divisor and quotient e.g. 	Think addition to solve subtraction with numbers up to 1000 e.g.  Think multiplication for division by grouping AND sharing e.g.  Make further connections between multiplication and division and fractions		

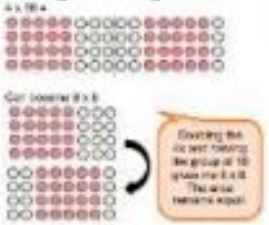
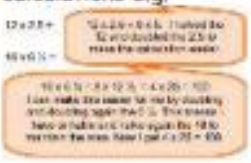
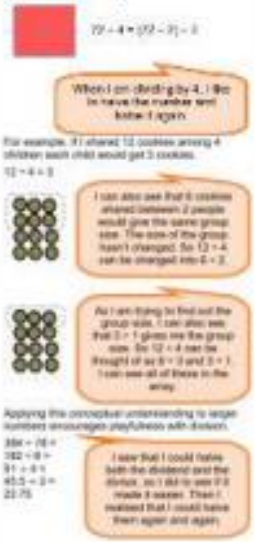
			 Make the connections between multiplication and division and fractions e.g. 			
Rounding				Identify the midpoint between a set of numbers under 1000 (and use this in order to understand rounding) e.g. 	Round to the nearest 10, 100, 1000 and other units such as money and time	Round to the nearest 10, 100, 1000, 10,000 and other units such as money, time, decimal numbers, negative numbers Use rounding as an estimation for multiplication and

						division e.g. 
Estimation			Estimate the distance of numbers from target numbers under 100 e.g. 	Estimate the distance of numbers from target numbers under 1000 to prepare for rounding e.g. 234 is 4 from 230 and 6 from 240, 240 is nearer to 230 than 240	Estimate the distance of numbers from target numbers up to 10,000, including tenths and hundredths to prepare for rounding e.g. 2134 is 34 from 2100 and 66 from 2200. 2134 is nearer to 2100 than 2200.	Estimate the distance of numbers from target numbers up to 1,000,000, including decimal numbers and negative numbers, to prepare for rounding e.g. 20,034 is 4 from 20,030 and 6 from 20,040. 20,034 is nearer to 20,030 than to 20,040.
Compensation			'Think 10', use benchmark numbers and compensate for trickier calculations e.g. 	Use benchmark numbers and compensate for adding and subtracting numbers up to 1000, including money e.g.  Use known facts to compensate for unknown multiplication	Use benchmark numbers and compensate for adding and subtracting numbers up to 10,000, including money and time e.g. "I could think of 2550 + 490 as compensation because adding 490 is like adding 500 and taking ten away. Now my calculation looks like this: 2550 + 500 - 10 = 3040. 	Use number knowledge to look for 'nearly numbers' in calculations (including decimals) e.g. 

				and division calculations up to 12x tables e.g. 	 Compensate in multiplication and division with numbers up to 1000 e.g. 	
Rebalancing			Understand the concept of equal sum (that the sum remains equal when the addends are rebalanced in addition) with numbers up to 20 e.g.  Understand the concept of equal difference (subtracting the same quantity from both subtrahend and minuend maintains the difference) with numbers up to 20 e.g. 	Use the equal sum concept with numbers up to 1000 as a method to solve addition calculations e.g.  Use the equal difference concept with numbers up to 1000 as a method to solve subtraction calculations e.g. 	Use the equal sum concept with numbers up to 10,000, including units of time and money, as a method to solve addition calculations e.g.  Use the equal difference concept with numbers up to 10,000, including units of time and money as a method to solve subtraction calculations	Apply the equal sum concept to a range of numbers and missing number problems (including units of time and money) e.g. 24 + □ = 30 + 3. 39 + 52 345 + 188 0.39 + 6.54 5.1 + 2.7 = □ + 4.8  Use the equal difference concept with a range of numbers, decimals and fractions e.g. 

					e.g.  Use equal difference concept with tenths and simple fractions e.g.	
X ÷ by powers of 10				Understand the concept of x and ÷ 10, and use known facts to multiply and divide by 10 and 100 e.g. 	Understand the concept of x and ÷ 10, and use known facts to multiply and divide by 10, 100 and 1000 e.g. 4000 x 6, 240 ÷ 4, 750mm = ____ cm.	X and ÷ by powers of 10, including 2 step problems. Includes decimals e.g.  "I know that 10 million has 8 place values columns and 7 0's. I know that 2 x 5 = 10. This already has one 0, therefore I could use 2000 x 5000 or 200 x 50,000"
Doubling and halving	Be able to find double of a number (up to 10) using concrete	Find doubles up to 20 and link this to repeated addition e.g. 	Find doubles and near doubles with numbers	Find doubles and near doubles with numbers up to 1000 e.g. 	Find doubles and near doubles with numbers up to 10,000. Use doubling and halving to find unknown products in multiples of 10 e.g. 9 x	

	resources e.g.  Be able to link 'fair sharing' between 2 to the language of 'equal parts' and 'half'	Understand the link between doubling, halving odds and evens	up to 100 e.g.  Relate repeated addition to multiplication e.g. 3 + 3 = 2 x 3 Use doubling to relate multiplication facts e.g.  Be able to generalise what happens when we halve a number that is an odd multiple of 10 e.g. 	Use doubling and halving to find unknown products e.g.  Be able to generalise what happens when we halve a number that is an odd multiple of 100 e.g. "If I halve 700, I can share three hundreds into each group but then I have to regroup the last hundred into ten tens. Each group will then get five tens. So three hundreds and 5 tens in each group is 350." "Where there is an odd multiple of ten or hundred, I will always have to regroup my last ten or hundred into the next place value column down. Then I will share halve of that between the two groups which will always be 5 units (5 ones or 5 tens)"	200 "I know that 9 x 100 = 900, and 200 is double 100, so to find the product I can double 900 to get 1800."	

Rearranging					Rearrange the multiplier and multiplicand using knowledge of doubles and halves to make a calculation more manageable e.g.  	Use the relationship between doubling and halving to manipulate the multiplicand and the multiplier to find more efficient calculations e.g.  Use halving and halving for division, understanding why the halving needs to be done to the dividend and divisor and not the quotient e.g.  Apply halving and doubling to fractions

						e.g. Pupils have all such 'wrong but correct' misunderstanding of this rule, for example: $5 \times c = 15 \times 7 = 28 \times 1$ Apply this understanding to fractions, for example: $\frac{1}{4} \times \frac{1}{5} =$ If we rewrite the first term and follow the second, we can transform the calculation to: $4 \times \frac{1}{4} = \frac{1}{5}$

Adding 1 | Bonds to 10 | Adding 10 | Bridging/ compensating

Adding 2 | Adding 0 | Doubles | Near doubles

Y1 facts

Y2 facts

+	0	1	2	3	4	5	6	7	8	9	10
0	0 + 0	0 + 1	0 + 2	0 + 3	0 + 4	0 + 5	0 + 6	0 + 7	0 + 8	0 + 9	0 + 10
1	1 + 0	1 + 1	1 + 2	1 + 3	1 + 4	1 + 5	1 + 6	1 + 7	1 + 8	1 + 9	1 + 10
2	2 + 0	2 + 1	2 + 2	2 + 3	2 + 4	2 + 5	2 + 6	2 + 7	2 + 8	2 + 9	2 + 10
3	3 + 0	3 + 1	3 + 2	3 + 3	3 + 4	3 + 5	3 + 6	3 + 7	3 + 8	3 + 9	3 + 10
4	4 + 0	4 + 1	4 + 2	4 + 3	4 + 4	4 + 5	4 + 6	4 + 7	4 + 8	4 + 9	4 + 10
5	5 + 0	5 + 1	5 + 2	5 + 3	5 + 4	5 + 5	5 + 6	5 + 7	5 + 8	5 + 9	5 + 10
6	6 + 0	6 + 1	6 + 2	6 + 3	6 + 4	6 + 5	6 + 6	6 + 7	6 + 8	6 + 9	6 + 10
7	7 + 0	7 + 1	7 + 2	7 + 3	7 + 4	7 + 5	7 + 6	7 + 7	7 + 8	7 + 9	7 + 10
8	8 + 0	8 + 1	8 + 2	8 + 3	8 + 4	8 + 5	8 + 6	8 + 7	8 + 8	8 + 9	8 + 10
9	9 + 0	9 + 1	9 + 2	9 + 3	9 + 4	9 + 5	9 + 6	9 + 7	9 + 8	9 + 9	9 + 10
10	10 + 0	10 + 1	10 + 2	10 + 3	10 + 4	10 + 5	10 + 6	10 + 7	10 + 8	10 + 9	10 + 10

Fluency also means automatic recall of key facts. These are the facts which children should be able to recall at the end of each year group. They will need to explicitly be taught strategies (doubles and near doubles, compensating etc.) to work them out as well as lots of practice in order to achieve automaticity in recall.

-	0	1	2	3	4	5	6	7	8	9	10
1											
2											
3											
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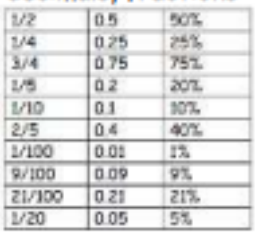
20											

By the end of an academic year, pupils should be able to recall the **Key Instant Recall Facts** for their year group quickly (within 5 seconds) and complete counting activities confidently and fluently. Children should also be able to recall the **Key Instant Recall Facts** for all previous year groups within the same amount of time.

Nursery	Reception	Year 1	Year 2
Recite the number names in order to 5. Touch count to 5. Use the language 'before', 'after', 'next' Sort objects and say which group is more/less. Name simple shapes Recite the number names to 10.	Name numbers in order to 10 and compare two numbers by saying which is more or less. Recognise quantities to 5 without counting up to 5 (subitise). To say one more than a given number up to 10. Be able to partition numbers to 5 in different combinations of two groups. Recall number bonds to 10 including partitioning facts. To know the days of the week in order. Recall names of numbers to 20. Know one more and 1 less for number 0-20 Identify 2D shapes: circle, square, triangle, rectangle, hexagon, pentagon Identify 3D shapes: sphere, cylinder, cone, cubes, cuboids Know number doubles up to 5 + 5.	Recite number names in order to 50 and beyond. To add one and two to any number. To know odd and even numbers to 20. Counts in 2's to 20. Count in 10's to 100. Count in 5's to 50. To know number bonds to 10. Facts within 10 as above and related subtraction facts Know o'clock and half past times. To add 10 to a number. To know doubles and halves of numbers to 10. To know near doubles to 5 Know the seasons in order Know the months of the year in order Identify 3D shapes: pyramids, square based pyramids	Recite the number names in order to 100. To know number bonds to 10 and 20. To know doubles and halves of numbers to 20. To know near doubles to 10. Count in 2's To know multiplication and division facts for the 2 times table. To use bridging and compensation for addition facts to 10 + 10. Count in 5's and 10's. To know multiplication and division facts for the 5 and 10 times table. Count in 3's to 36. To tell the time for o'clock, quarter past, half past and quarter to as well as intervals of 5. To know multiplication and division facts for the 3 times table. Facts within 20 as above and related subtraction facts Number of minutes in an hour; number of hours in a day Coin recognition up to £2 and note recognition Know 100p=£1 Identify 2D shapes: quadrilaterals, regular and irregular polygons Identify 3D shapes: cuboids, prisms

Year 3	Year 4	Year 5	Year 6
To know the number bonds for all numbers up to 20. Count in 50's and 100's. Count in 3's. To know multiplication and division facts for the 3 times table. Count in 4's. To know multiplication and division facts for the 4 times table. Count up and down in tenths. Recognise decimal equivalents of tenths. Count in 8's. To know multiplication and division facts for the 8 times table. Recall facts about durations of time (seconds in a minute, minutes in an hour, hours in a day). To multiply and divide 1 digit numbers by 10. Sums and differences between pairs of numbers which are multiples of 10 and 100. Doubles and halves of multiples of 10 or 100. Complements to 100. Complements to 60 (time). Complements of fractions with the same denominator that make 1 e.g. 3/7 + 4/7 = 1 Number of days in each month and in a year including a leap year. Recognise right angles. Recognise parallel and perpendicular lines. Recognise horizontal and vertical.	To know number bonds to 100. Count in 25's and 1000's. Count in 6's. To know multiplication and division facts for the 6 times table. Count in 9's and 11's. To know multiplication and division facts for the 9 and 11 times table. Count in 7's and 12's. To know multiplication and division facts for the 7 and 12 times table. To recognise decimal equivalents of fractions (1/2, ¼, ¾, tenths and hundredths). Multiply and divide 1 and 2 digit numbers by 10 and 100. Doubles and halves of multiples of 10, 100 or 1000 (6 + 6, 60 + 60, 600 + 600, 6000 + 6000). Multiplication and division by zero and one facts. Conversion of kilometres to metres, hours to minutes, years to months, weeks to days. Complements of tenths that make 1. Complements of hundredths that make 1. Convert between decimals and fractions for ½, ¼, ¾ and any number of tenths and hundredths . Read roman numerals to 100. Know right angles = 90 degrees. Know the types of triangle (isosceles, equilateral, scalene).	Identify prime numbers up to 20 (2, 3, 5, 7, 11, 13, 17, 19). Recall metric conversions (1 kilogram = 1000 grams, 1 kilometre = 1000 metres, 1 metre = 100 centimetres, 1 metre = 1000 millimetres, 1 centimetre = 10 millimetres 1 litre = 1000 millilitres). Recall square numbers up to 12 squared and their square roots. Read Roman numerals to 1000. Know angles on a straight line = 180 degrees. Know angles in a triangle = 180 degrees. Know angles around a point = 360 degrees.	Convert between decimals, fractions and percentages  Identify prime numbers up to 50 (2, 3, 5, 7, 11, 13, 17, 19, 23, 27, 29, 31, 37, 41, 43, 47). Illustrate and name parts of a circle, including radius, diameter and circumference and know that the diameter is twice the radius.

1/2	0.5	50%
1/4	0.25	25%
3/4	0.75	75%
1/5	0.2	20%
1/10	0.1	10%
2/5	0.4	40%
1/100	0.01	1%
9/100	0.09	9%
21/100	0.21	21%
1/20	0.05	5%

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This policy has been adapted from the Fluency Progression document from Portland Spencer Academy